

## PRISMATIC $F$ -GAUGES

(MCM–YMSC  $p$ -ADIC GEOMETRY LEARNING SEMINAR, SPRING 2023)

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**Purpose:** The main goal of this semester is to understand the ideas of geometrization in  $p$ -adic geometry and discuss prismatic  $F$ -gauges by following Bhatt’s lecture notes. The last few lectures are reserved to quickly learn another topic (most likely,  $\mathcal{D}$ -modules on rigid analytic spaces).

**Time:** 2:00-3:30 pm on Mondays

**Location:** MCM 110

**Live Streaming:** We plan to offer a live streaming. However, to maintain a relaxing learning atmosphere, the Zoom link will not be posted on the websites, and we do not record the talks.

**Website:** <http://shizhang.li/seminars/prismatic%20F-gauges%20and%20D-mod%20separately.html>

**Mailing List:** We make the seminar announcements via the mailing list. To join the mailing list, please contact Shizhang.

**Schedule:** The following is an outline and suggestion for each talk. Sometimes, too many topics are assigned to one talk, and/or relevant remarks in [Bha] may not have been covered in the previous talks. Please reorganize the materials to give a 90-minute-long *comprehensible* talk, rather than copying the lecture notes. The organizers, especially SL, are happy to discuss the materials and organization of the lectures. Please feel free to contact us. Junior speakers are encouraged to talk with us during the lecture preparation.

**Lecture 1.** *Filtrations and endomorphisms.* (2/20, Koji)

Give a short introduction based on [Bha, §1.3-1.4] and then discuss [Bha, §2.1-2.2].

**Lecture 2.** *The de Rham cohomology in characteristic 0.* (2/27, Yupeng)

Discuss [Bha, §2.3-2.4]. See [Dri20, § 1.3] for the notion of cones and quasi-ideals. Note that some of the remarks (e.g. Remark 2.2.15) [Bha, §2.2] may not have been covered in Lecture 1. Explain them if necessary.

**Lecture 3.** *The de Rham cohomology of  $p$ -adic formal schemes.* (3/6, Shizhang)

Discuss [Bha, §2.5-2.6]. Introduce objects in Definitions 2.5.1 and 2.5.3, and cover Theorem 2.5.6, Corollary 2.5.10, Corollary 2.6.8, and Corollary 2.6.11. Please decide how much preliminaries and proofs you want to present as well as how many additional remarks on Bhatt’s notes that you want to explain according to the time constraint.

**Lecture 4.** *The conjugate filtration.* (3/13, Shizhang)

Discuss [Bha, §2.7-2.8]. Cover Corollary 2.7.2, Constructions 2.7.7 & 2.7.8, Theorem 2.7.9, Construction 2.7.11, Proposition 2.7.12, Corollary 2.7.14, and almost all materials in §2.8.

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Obviously there are a lot of materials to cover, so we suggest you mention these constructions together and they are of a similar flavor. One way to save some time, for instance, is to note the similarities between 2.7.7-9 and 2.7.11-12, so you could focus on one set and only state the other set without further comments. Maybe 2.7.14 should come in the end (but do make sure that you cover it!) as a mind boggling application of the above rather abstract constructions and their properties.

**Lecture 5.** *Prismatization over  $k$ .* (3/20, Shanwen)

Discuss [Bha, §3.1-3.3]. Please cover §3.1 and §3.3. You can recall/admit materials in §3.2 for some discussions in §3.3.

**Lecture 6.** *Gauges over  $k$ .* (3/27, Daxin)

Discuss [Bha, §3.4-3.5]. The main result is Theorem 3.5.1. We suggest you use it to motivate the discussions in these two subsections. So the lecture can be a very long proof of Mazur's theorem with a lot of aside constructions and properties and discussions. . . .

**Lecture 7.** *Syntomification in characteristic  $p$ .* (4/3, Yichao)

Discuss [Bha, §4.1-4.3]. Introduce Definitions 4.1.1 & 4.2.1, and mention (some of) their properties buried in the remarks; then discuss Example 4.2.6, Proposition 4.3.1, Example 4.3.3, Definition 4.3.4. Fill the remaining time with other materials in these sections.

**Lecture 8.** *Syntomic cohomology in characteristic  $p$ .* (4/10, Yong Suk)

Discuss [Bha, §4.4-4.5]. One reasonable plan is to focus on §4.5 and use it as motivation to introduce materials in §4.4.

**Lecture 9.** *Prismatization.* (4/17, Jiedong)

Discuss [Bha, §5.1-5.2]. Please arrange your talk to cover materials in §5.1 and 5.2.1-5.2.4.

**Lecture 10.** *Filtered prismatization.* (4/24, Jiahao)

Discuss [Bha, §5.3-5.5]. Main focus for this lecture is on §5.3. You can mention results in §5.4, and only sketch key proof ideas. Here is our suggestion on §5.5: you can read up to 5.5.11 and then try to read 5.5.19-20. Since 5.5.11 will be summoned in later talks, it would be good if you can write down its statement (and related statements before it, such as 5.5.6-5.5.10). Then try to explain these materials (namely, the meaning of symbols of the statement and/or proof ideas) when  $R$  is a perfectoid or something like  $R = R_0\langle X^{1/p^\infty} \rangle/(X)$  where  $R_0$  is perfectoid. (For the sake of concreteness and saving some time, it might be a good idea to replace quasi-syntomic with smooth or lci or perfectoid or something like  $R$  above, and qrsp can be replaced with the latter two classes of rings.)

**Lecture 11.** *Syntomification.* (5/8, Heng)

Discuss [Bha, §6.1-6.3]. It will be better to focus on §6.3 and mention concepts and statements in §6.1 & 6.2 when necessary. You also need to recall a lot of materials from previous lectures for the audience.

**Lecture 12.** *Syntomic cohomology and Galois cohomology.* (5/15, )

Discuss [Bha, §6.4-6.5]: §6.4 is a reward for those audience hanging so far. As for §6.5, explain some important points instead of trying to cover everything. Alternatively, you may give an overview of [Bha, §6.4-6.7].

The rest of the seminar will focus on another topic.

**Lecture 13.** . (5/22, )

**Lecture 14.** . (5/29, )

**Lecture 15.** . (6/5, )

#### REFERENCES

- [Bha] Bhargav Bhatt, *Prismatic  $F$ -gauges*, available at <https://www.math.ias.edu/~bhatt/teaching/mat549f22/lectures.pdf>.  
[Dri20] Vladimir Drinfeld, *Prismatization*, May 2020, p. arXiv:2005.04746.